On the Perturbative Evaluation of Thermal Green's Functions in the Bulk and Shear Channels of Yang-Mills Theory

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Outline

- Motivation
- Correlators in SU(Nc) Yang-Mills theory
- Results
  - Correlators in UV
  - Spectral densities
  - Discussion on HTL correction
- Summary and Outlook

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Linearized Viscous Hydrodynamics

- Hydrodynamic with small viscosity turns out to be a successful theory for the description of QGP in high energy HIC!

- Macroscopic Form of Energy Momentum Tensor:

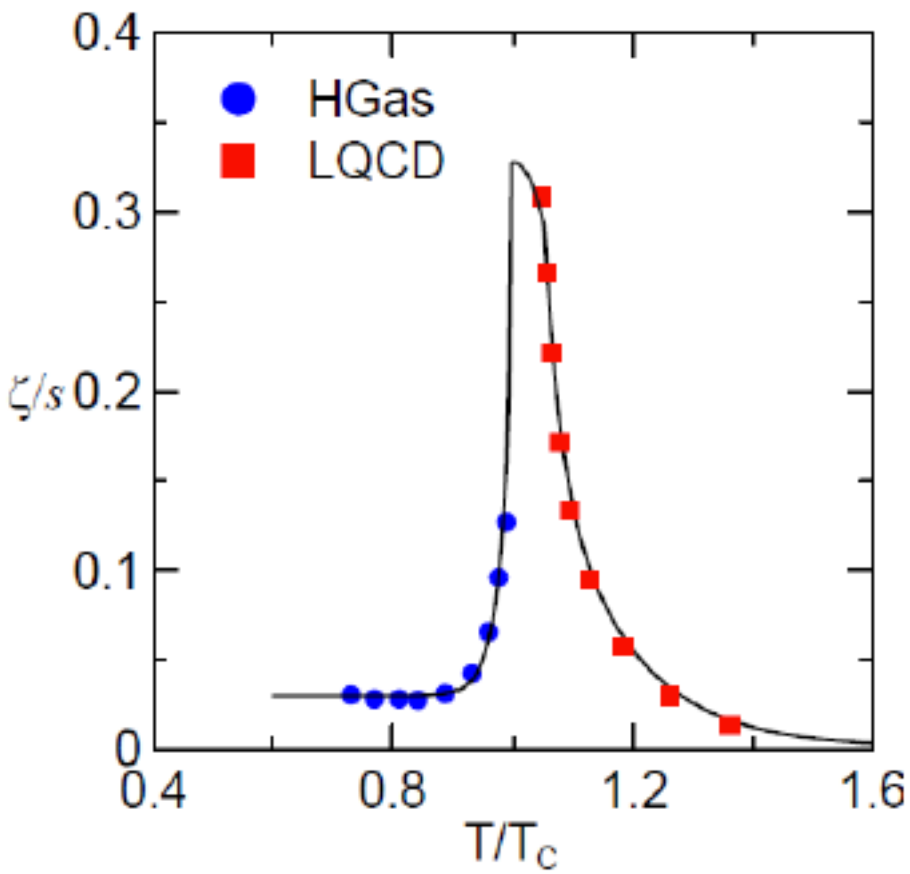
$$T^{\mu\nu} = -Pg^{\mu\nu} + (e+P)u^{\mu}u^{\nu} + \Delta T^{\mu\nu}$$

$$\Delta T^{\mu\nu} = \eta(\Delta^{\mu}u^{\nu} + \Delta^{\nu}u^{\mu}) + (\frac{2}{3}\eta - \zeta)H^{\mu\nu}\partial_{\rho}u^{\rho}$$

$$\Delta^{\mu} = \partial_{\mu} - u_{\mu}u^{\beta}\partial_{\beta}, \quad H^{\mu\nu} = u^{\mu}u^{\nu} - g^{\mu\nu}$$

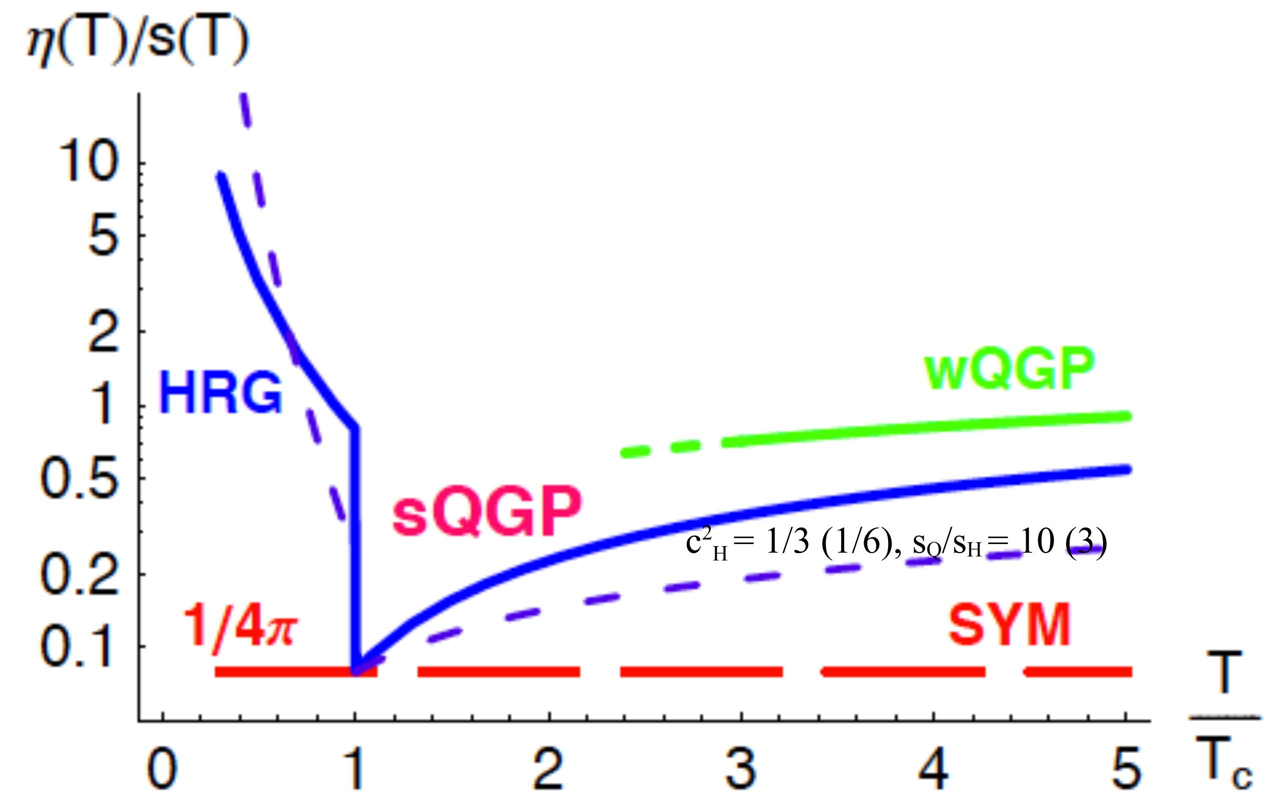
Bulk and Shear Viscosities

Bulk Viscosity



Karsh&Kharzeev&Tuchin, 0711.0914
Noronha&Noronha&Greiner, 0811.1571

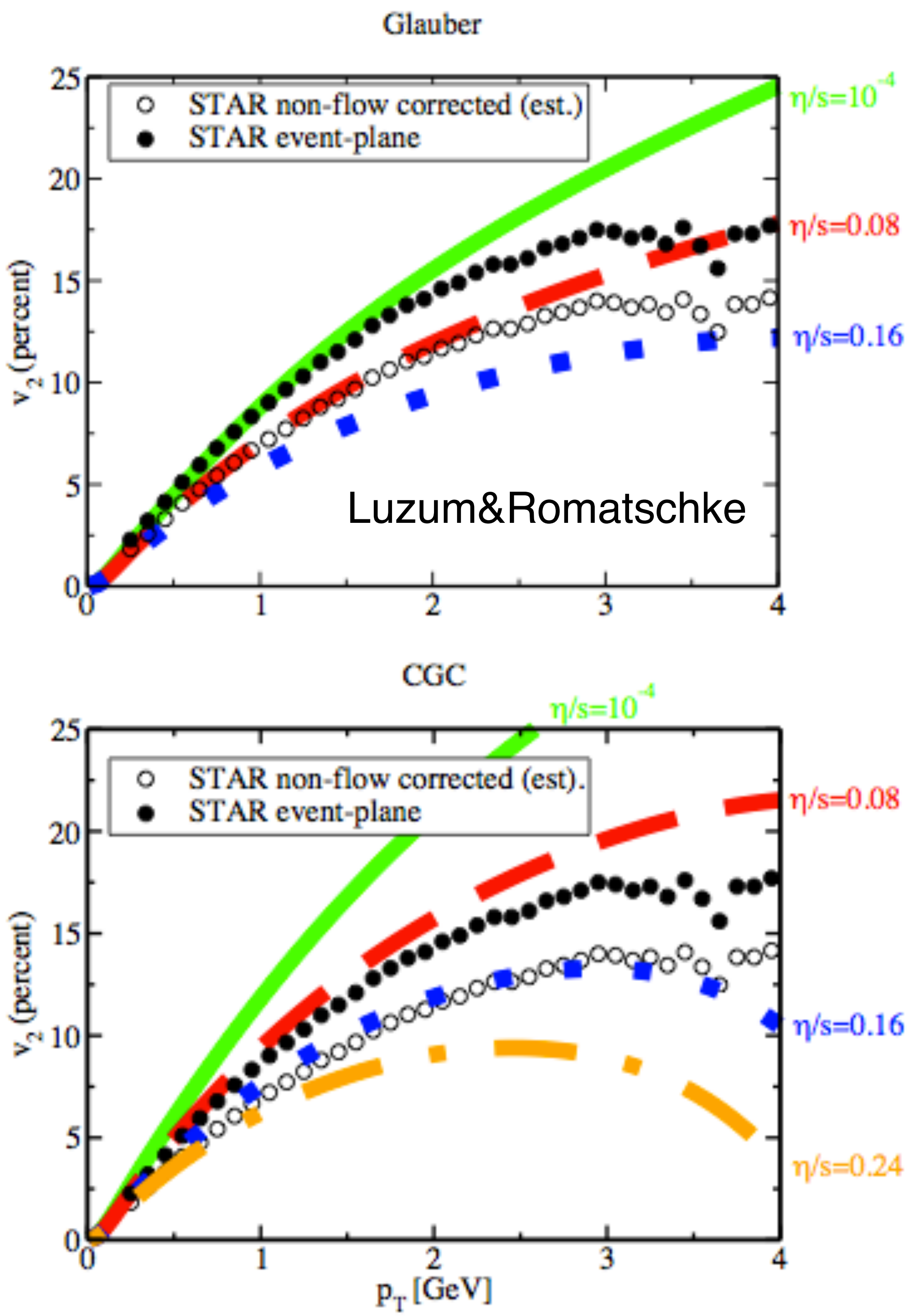
Shear Viscosity



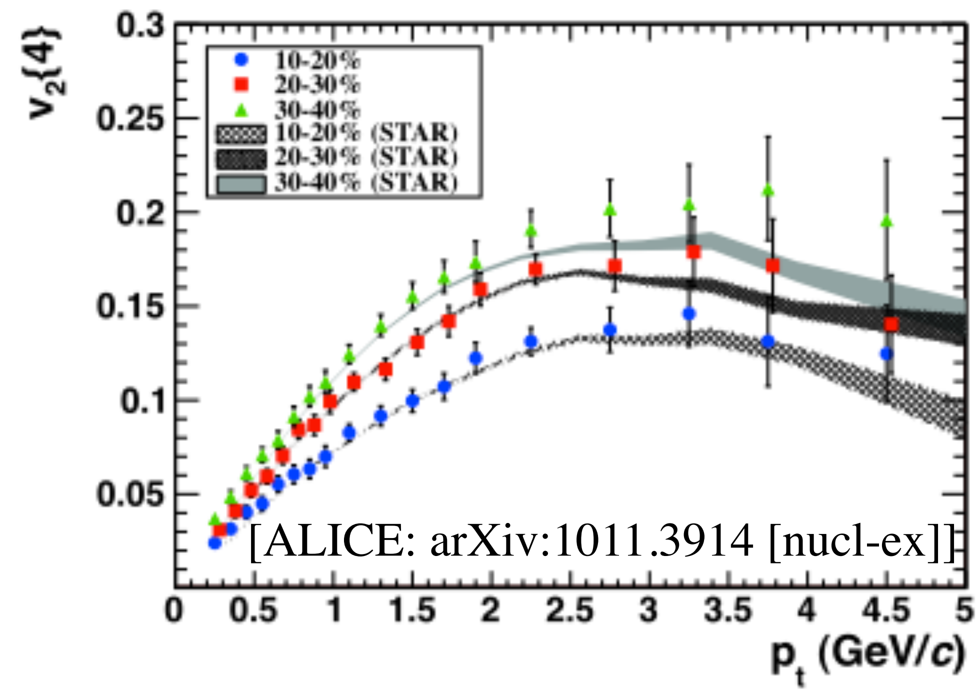
Hirano&Gyulassy, nucl-th/0506049

What about characteristics of QGP in HIC?

Puzzles from HIC



$\eta/s \sim 0.08 - 0.16$



- What are η, ζ,... in QCD? Is the plasma 'strongly coupled'? Is N = 4 SYM really a good model for QGP?
- Ultimate answer only from non-perturbative calculations in QCD!

Bulk and shear viscosities: Kubo formulae

- Matching of linearized hydrodynamic and linear response description in QFT---Kubo formulae: Viscosities and other transport coeffs. are obtainable from retarded Minkowskian correlators of energy momentum tensor

$$\eta = \pi \lim_{\omega \to 0} \frac{\rho_{12,12}(\omega, \mathbf{k} = \mathbf{0})}{\omega}$$

$$\zeta = \frac{\pi}{9} \sum_{i,j=1}^{3} \lim_{\omega \to 0} \frac{\rho_{ii,jj}(\omega, \mathbf{k} = \mathbf{0})}{\omega}$$

$$\rho_{\mu\nu\rho\sigma} = \mathrm{Im} G^R_{\mu\nu\rho\sigma}(\omega, \mathbf{0})$$

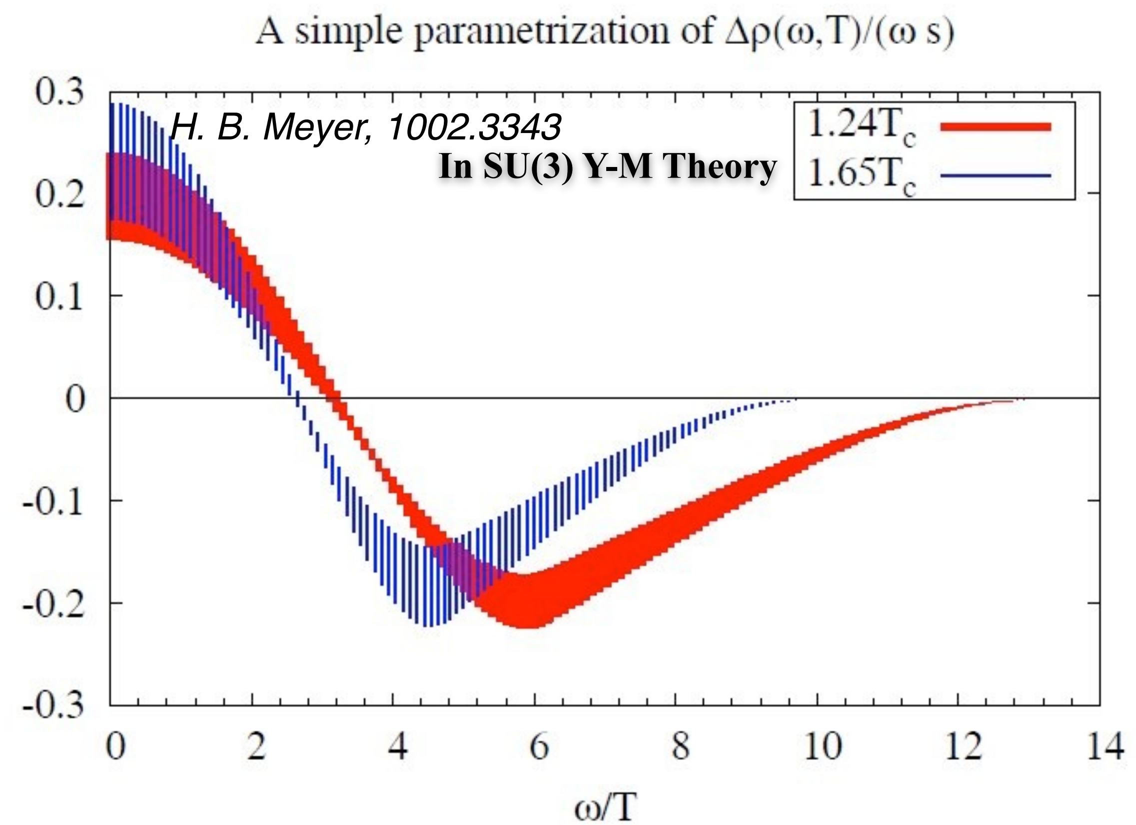
$$G^R_{\mu\nu\rho\sigma}(\omega, \mathbf{0}) \equiv i \int_0^\infty dt e^{i\omega t} \int d^3x \, \langle [T_{\mu\nu}(t, \mathbf{x}), T_{\rho\sigma}(0, \mathbf{0})] \rangle$$

$$G_R(\omega) = \tilde{G}_E(p_n \to -i[\omega + i0^+], \mathbf{0})$$

Viscosities from the lattice

- Lattice determines spectral density ρ from Euclidean correlators: Need to invert

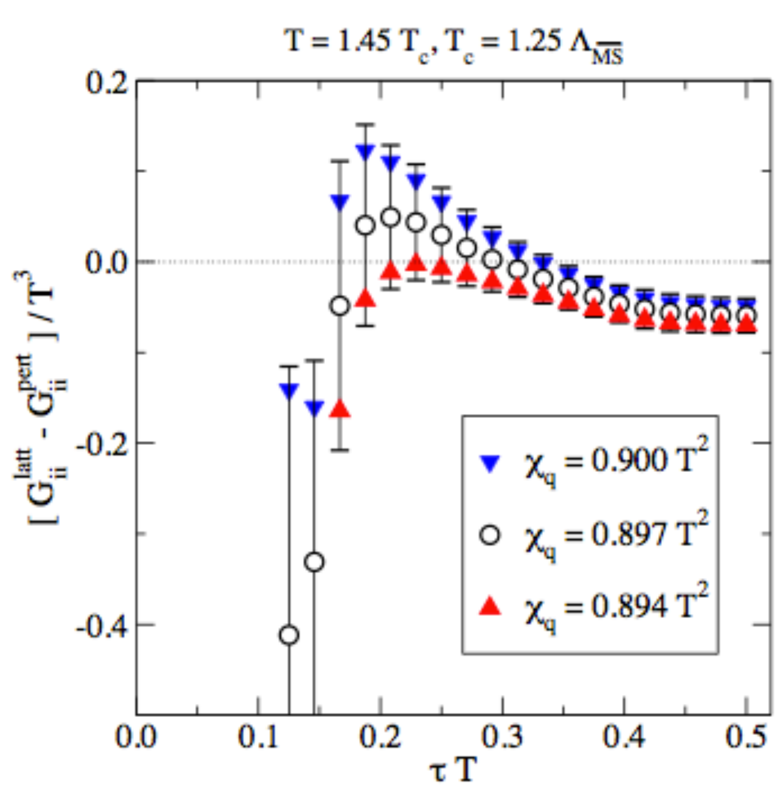
$$G(\hat{\tau}) = \int_0^\infty \frac{d\omega}{\pi} \rho(\omega) \frac{\cosh\left(\frac{1}{2} - \hat{\tau}\right)\beta\omega}{\sinh\frac{\beta\omega}{2}}$$

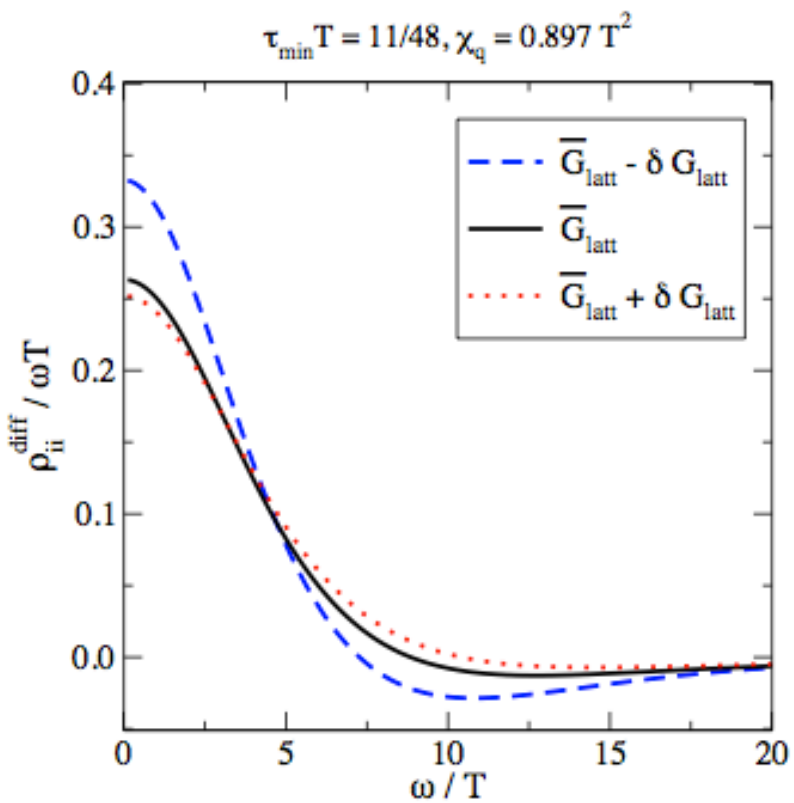


- For extracting IR limit of ρ, need to understand its behavior also at $\omega \gtrsim \pi T$ — very non-trivial challenge for lattice QCD, requiring perturbative input!

Successful application of pQCD result

- For the vector-current correlator, 5-loop vacuum limit and accurate lattice data available $\Rightarrow$ Model-independent analytic continuation of Euclidean correlator à la [Burnier, Laine, Mether; EPJC 71] possible
- Result: Estimate for flavor current spectral density and flavor diffusion coefficient [Burnier, Laine; EPJC 72] $2\pi TD \gtrsim 0.8$





$$G_{ii}(\tau T) = \chi_q T + G_V(\tau T) \qquad G_V(\tau) \equiv -\sum_{\mu=0}^{3} \int_{\mathbf{x}} \left\langle (\bar\psi\gamma_\mu\psi)(\tau,\mathbf{x})\,(\bar\psi\gamma^\mu\psi)(0,\mathbf{0}) \right\rangle_T$$

Setup

- SU(Nc) YM theory

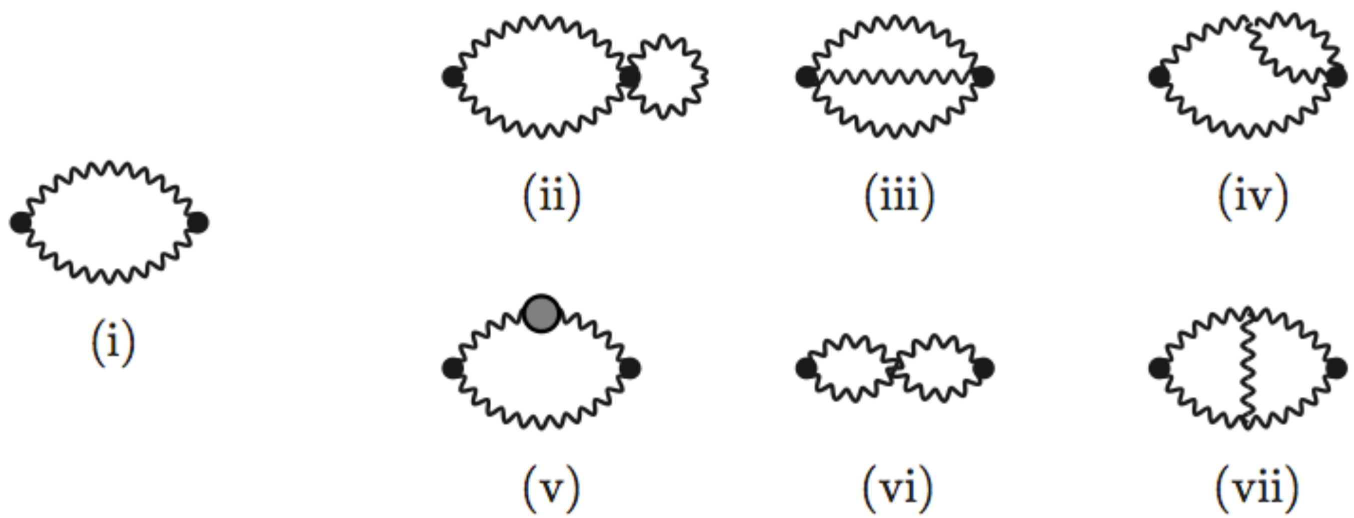
$$S_E = \int_0^\beta \mathrm{d}\tau \int \mathrm{d}^{3-2\epsilon}\mathbf{x} \left\{ \frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} \right\}$$

- Define:
  - $G_\theta(x) \equiv \langle \theta(x)\theta(0)\rangle_\mathrm{c}\ , \quad \theta \equiv c_\theta\, g_\mathrm{B}^2 F^a_{\mu\nu} F^a_{\mu\nu}$
  - $G_\chi(x) \equiv \langle \chi(x)\chi(0)\rangle\ , \quad \chi \equiv c_\chi\, \epsilon_{\mu\nu\rho\sigma} g_\mathrm{B}^2 F^a_{\mu\nu} F^a_{\rho\sigma}$
  - $G_\eta(x) = -16 c_\eta^2 \,\langle T_{12}(x)\, T_{12}(0)\rangle_c\,.$

  where $T_{\mu\nu} = \frac{1}{4}\delta_{\mu\nu} F^a_{\alpha\beta} F^a_{\alpha\beta} - F^a_{\mu\alpha} F^a_{\nu\alpha}\,,$

Correlators to NLO

The LO and NLO Feynman graphs contributing to the correlators



- Write down diagrammatic expansions for Euclidean correlators in momentum space $\tilde{G}_\alpha(P) \equiv \int_x e^{-iP\cdot x}\tilde{G}_\alpha(x)$
- Carry out Matsubara sums by 'cutting' thermal lines and evaluate remaining 3d integrals at high P to get the OPE
- Extract the spectral densities with $\rho(\omega) = \mathrm{Im}\left[\tilde{G}(P)\right]_{P\to(-i[\omega+i0^+],\mathbf{0})}$.

Correlators in the bulk channel

$$
\begin{aligned}
\frac{\tilde{G}_\theta(P)}{4d_A c_\theta^2} &= g_{\rm B}^4 (D-2)\left[-\mathcal{J}_{\rm a}^0 + \frac{1}{2}\,\mathcal{J}_{\rm b}^0\right] \\
&+ \quad g_{\rm B}^6 N_{\rm c}\Big\{2(D-2)\left[-(D-2)\mathcal{I}_{\rm a}^0 + (D-4)\mathcal{I}_{\rm b}^0\right] + (D-2)^2\left[\mathcal{I}_{\rm c}^0 - \mathcal{I}_{\rm d}^0\right] \\
&\quad + \frac{22-7D}{3}\mathcal{I}_{\rm f}^0 - \frac{(D-4)^2}{2}\mathcal{I}_{\rm g}^0 + (D-2)\left[-3\mathcal{I}_{\rm e}^0 + 3\mathcal{I}_{\rm h}^0 + 2\mathcal{I}_{\rm i}^0 - \mathcal{I}_{\rm j}^0\right]\Big\}\,, \qquad (3.14) \\
\frac{\tilde{G}_\chi(P)}{-16 d_A c_\chi^2 (D-3)} &= g_{\rm B}^4 (D-2)\left[-\mathcal{J}_{\rm a}^0 + \frac{1}{2}\,\mathcal{J}_{\rm b}^0\right] \\
&+ \quad g_{\rm B}^6 N_{\rm c}\Big\{2(D-2)\left[-\mathcal{I}_{\rm a}^0 (D-4)\mathcal{I}_{\rm b}^0\right] + (D-2)^2\left[\mathcal{I}_{\rm c}^0 - \mathcal{I}_{\rm d}^0\right] \qquad (3.15) \\
&\quad - \frac{2D^2-17D+42}{3}\mathcal{I}_{\rm f}^0 - 2(D-4)\mathcal{I}_{\rm g}^0 + (D-2)\left[-3\mathcal{I}_{\rm e}^0 + 3\mathcal{I}_{\rm h}^0 + 2\mathcal{I}_{\rm i}^0 - \mathcal{I}_{\rm j}^0\right]\Big\}\,,
\end{aligned}
$$

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Correlators in the shear channel

$$
\begin{aligned}
\frac{\tilde{G}_\eta(P)}{4d_A c_\eta^2 \Lambda^{2\epsilon}} = {} & \frac{D(D-2)(D-3)}{8}\left(2\mathcal{J}_{\mathrm{a}}^0 - \mathcal{J}_{\mathrm{b}}^0\right) - (D-2)(D-3)\mathcal{J}_{\mathrm{b}}^2 \\
+ \; & D(D-3)\left(\mathcal{J}_{\mathrm{a}}^1 - \mathcal{J}_{\mathrm{b}}^1\right) \\
+ \; & g_{\mathrm{B}}^2 N_{\mathrm{c}} \Bigg\{ \frac{D(D-2)(D-3)}{4}\Bigg(2\mathcal{I}_{\mathrm{a}} + 4\mathcal{I}_{\mathrm{b}}^1 + 2\mathcal{I}_{\mathrm{b}}^2 + 4\mathcal{I}_{\mathrm{d}}^1 + 12\mathcal{I}_{\mathrm{d}}^2 + 2\mathcal{I}_{\mathrm{e}}^0 \\
+ \; & \mathcal{I}_{\mathrm{e}}^2 + 4\mathcal{I}_{\mathrm{e}}^3 - 3\mathcal{I}_{\mathrm{f}}^1 - 4\mathcal{I}_{\mathrm{h}}^0 - 4\mathcal{I}_{\mathrm{h}}^1 + \mathcal{I}_{\mathrm{h}}^3 - 4\mathcal{I}_{\mathrm{i}}^1 - 4\mathcal{I}_{\mathrm{i}}^2 - 2\mathcal{I}_{\mathrm{i}}^3 + \mathcal{I}_{\mathrm{j}}^0\Bigg) \\
+ \; & \frac{D(D-2)}{2}\Bigg(-\mathcal{I}_{\mathrm{e}}^4 - 2\mathcal{I}_{\mathrm{e}}^5 + 4\mathcal{I}_{\mathrm{e}}^6 + 2\mathcal{I}_{\mathrm{e}}^7 + 2\mathcal{I}_{\mathrm{h}}^4 + \mathcal{I}_{\mathrm{h}}^5 - 4\mathcal{I}_{\mathrm{h}}^6 - 2\mathcal{I}_{\mathrm{h}}^7\Bigg) \\
- \; & \frac{(D-2)^2(D-3)}{4}\Bigg(D\mathcal{I}_{\mathrm{c}} - D\mathcal{I}_{\mathrm{d}}^0 - 8\mathcal{I}_{\mathrm{d}}^3\Bigg) + D(D-3)\Bigg(-4\mathcal{I}_{\mathrm{h}}^2 + 2\mathcal{I}_{\mathrm{j}}^1 + \mathcal{I}_{\mathrm{j}}^2\Bigg) \\
- \; & D(D-6)\Bigg(\mathcal{I}_{\mathrm{j}}^5 + 2\mathcal{I}_{\mathrm{j}}^6\Bigg) + \frac{12-16D+3D^2}{2}\Bigg(2\mathcal{I}_{\mathrm{j}}^3 + \mathcal{I}_{\mathrm{j}}^4\Bigg) \\
+ \; & \frac{D(D-3)(3D-10)}{4}\mathcal{I}_{\mathrm{e}}^1\Bigg\}\,, \qquad (3.16)
\end{aligned}
$$

Wilson coefficients for OPE

- In UV, define $\Delta\tilde{G}_\alpha(P) \equiv \tilde{G}_\alpha(P) - \tilde{G}_\alpha^{T=0}(P)$

$$
\begin{aligned}
\frac{\Delta\tilde{G}_\theta(P)}{4c_\theta^2 g^4} &= \frac{3}{P^2}\left(\frac{p^2}{3}-p_n^2\right)\left[1+\frac{g^2N_c}{(4\pi)^2}\left(\frac{22}{3}\ln\frac{\bar{\mu}^2}{P^2}+\frac{203}{18}\right)\right](e+p)(T) \\
&- \frac{2}{g^2b_0}\left[1+g^2b_0\ln\frac{\bar{\mu}^2}{\zeta_\theta P^2}\right](e-3p)(T)+\mathcal{O}\left(g^4,\frac{1}{P^2}\right) \\
\frac{\Delta\tilde{G}_\chi(P)}{-16c_\chi^2 g^4} &= \frac{3}{P^2}\left(\frac{p^2}{3}-p_n^2\right)\left[1+\frac{g^2N_c}{(4\pi)^2}\left(\frac{22}{3}\ln\frac{\bar{\mu}^2}{P^2}+\frac{347}{18}\right)\right](e+p)(T) \\
&+ \frac{2}{g^2b_0}\left[1+g^2b_0\ln\frac{\bar{\mu}^2}{\zeta_\chi P^2}\right](e-3p)(T)+\mathcal{O}\left(g^4,\frac{1}{P^2}\right) \\
\frac{\Delta\tilde{G}_\eta(P)}{4c_\eta^2} &= -\left\{1+\frac{3}{P^2}\left(\frac{p^2}{3}-p_n^2\right)-\frac{1}{3}\frac{g^2N_c}{(4\pi)^2}\left[22+\frac{41}{P^2}\left(\frac{p^2}{3}-p_n^2\right)\right]\right\}(e+p)(T) \\
&+ \frac{4}{3g^2b_0}\left[1-g^2b_0\ln\zeta_\eta\right](e-3p)(T)+\mathcal{O}\left(g^4,\frac{1}{P^2}\right)
\end{aligned}
$$

Spectral functions

$$\rho(\omega) = \mathrm{Im}\left[\tilde{G}(P)\right]_{P\to(-i[\omega+i0^+],\mathbf{0})} .$$

- After Matsubara sums, the imaginary part can be extracted with

$$\frac{1}{\omega \pm i0^+} = \mathbb{P}\left(\frac{1}{\omega}\right) \mp i\pi\delta(\omega)$$

- Example:

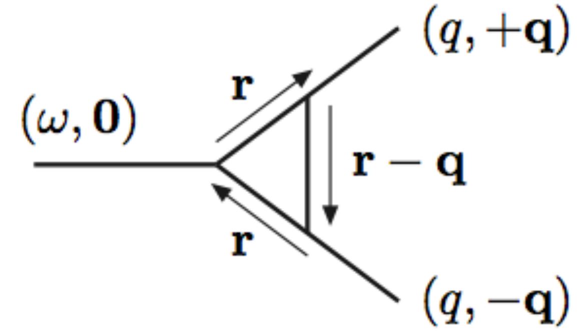
$$\mathcal{I}_{\mathrm{j}}^{0}(P) \equiv \sum\!\!\!\!\!\!\!\int_{Q,R} \frac{P^6}{Q^2R^2[(Q-R)^2+\lambda^2](Q-P)^2(R-P)^2}$$

Denoting $E_q \equiv q\ , \quad E_r \equiv r\ , \quad E_{qr} \equiv \sqrt{(\mathbf{q}-\mathbf{r})^2+\lambda^2}\ ,$

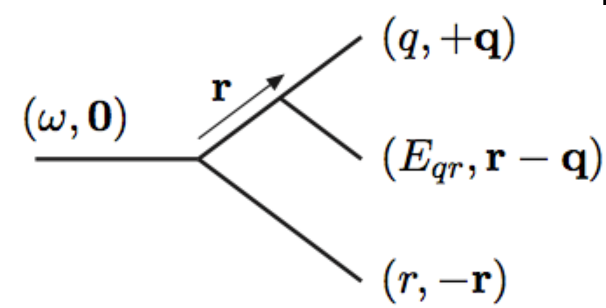
$\rho_{\mathcal{I}_j^0}$

$$
\begin{aligned}
\rho_{\mathcal{I}_j^0}(\omega) = \int_{\mathbf{q},\mathbf{r}} \frac{\omega^6 \pi}{4 q r E_{qr}} \Bigg\{ & \frac{1}{8q^2}\Big[\delta(\omega - 2q) - \delta(\omega + 2q)\Big] \times \\
& \times \Bigg[\left(\frac{1}{(q + r - E_{qr})(q + r)} - \frac{1}{(q - r + E_{qr})(q - r)} \right)(1 + 2n_q)(n_{qr} - n_r) \\
& + \left(\frac{1}{(q + r + E_{qr})(q + r)} - \frac{1}{(q - r - E_{qr})(q - r)} \right)(1 + 2n_q)(1 + n_{qr} + n_r) \Bigg] \\
+ & \frac{1}{8r^2}\Big[\delta(\omega - 2r) - \delta(\omega + 2r)\Big] \times \\
& \times \Bigg[\left(\frac{1}{(q + r - E_{qr})(q + r)} - \frac{1}{(q - r - E_{qr})(q - r)} \right)(1 + 2n_r)(n_{qr} - n_q) \\
& + \left(\frac{1}{(q + r + E_{qr})(q + r)} - \frac{1}{(q - r + E_{qr})(q - r)} \right)(1 + 2n_r)(1 + n_{qr} + n_q) \Bigg] \\
+ & \Big[\delta(\omega - q - r - E_{qr}) - \delta(\omega + q + r + E_{qr})\Big] \frac{(1 + n_{qr})(1 + n_q + n_r) + n_q n_r}{(q + r + E_{qr})^2 (q - r + E_{qr})(q - r - E_{qr})} \\
+ & \Big[\delta(\omega - q - r + E_{qr}) - \delta(\omega + q + r - E_{qr})\Big] \frac{n_{qr}(1 + n_q + n_r) - n_q n_r}{(q + r - E_{qr})^2 (q - r + E_{qr})(q - r - E_{qr})} \\
+ & \Big[\delta(\omega - q + r - E_{qr}) - \delta(\omega + q - r + E_{qr})\Big] \frac{n_r(1 + n_q + n_{qr}) - n_q n_{qr}}{(q - r + E_{qr})^2 (q + r + E_{qr})(q + r - E_{qr})} \\
+ & \Big[\delta(\omega + q - r - E_{qr}) - \delta(\omega - q + r + E_{qr})\Big] \frac{n_q(1 + n_r + n_{qr}) - n_r n_{qr}}{(q - r - E_{qr})^2 (q + r + E_{qr})(q + r - E_{qr})} \Bigg\}
\end{aligned}
$$

Factorized int./
Virtual correction



Phase space int./
Real correction



Virtual corrections

$$\rho^{(\mathrm{fz})}_{\mathcal{I}^0_{\mathrm{j}}}(\omega) = \int_{\mathbf{q},\mathbf{r}} \frac{\omega^6\pi}{4qrE_{qr}} \times$$

$$\times \frac{1}{8r^2}\Big[\delta(\omega-2r) - \delta(\omega+2r)\Big] \times \quad \omega > 0$$

$$\times \Bigg[\left(\frac{1}{(q+r-E_{qr})(q+r)} - \frac{1}{(q-r-E_{qr})(q-r)}\right)(1+2n_r)(n_{qr}-n_q)$$

$$+\left(\frac{1}{(q+r+E_{qr})(q+r)} - \frac{1}{(q-r+E_{qr})(q-r)}\right)(1+2n_r)(1+n_{qr}+n_q)$$

- $\rho^{(\mathrm{fz,p})}_{\mathcal{I}^0_{\mathrm{j}}}(\omega) \approx \frac{\omega^4}{(4\pi)^3}(1+2n_{\frac{\omega}{2}}) \int_\lambda^{\frac{\omega}{2}} \frac{\mathrm{d}q}{q} \ln\left|\frac{q+\sqrt{q^2-\lambda^2}}{q-\sqrt{q^2-\lambda^2}}\right| .$
- $\rho^{(\mathrm{fz,e})}_{\mathcal{I}^0_{\mathrm{j}}}(\omega) = \frac{\omega^4}{(4\pi)^3}(1+2n_{\frac{\omega}{2}})\Bigg\{$

$$\int_0^\infty \mathrm{d}q\, n_q\, \mathbb{P}\left[\frac{1}{q+\frac{\omega}{2}}\ln\left|\frac{\lambda^2}{2q\omega-\lambda^2}\right| + \frac{1}{q-\frac{\omega}{2}}\ln\left|\frac{\lambda^2}{2q\omega+\lambda^2}\right|\right]$$

$$+ \int_\lambda^\infty \mathrm{d}q\, n_q\left[\frac{1}{q}\ln\left|\frac{q+\frac{\lambda^2}{\omega}+\sqrt{q^2-\lambda^2}}{q+\frac{\lambda^2}{\omega}-\sqrt{q^2-\lambda^2}}\right| + \frac{1}{q}\ln\left|\frac{q-\frac{\lambda^2}{\omega}+\sqrt{q^2-\lambda^2}}{q-\frac{\lambda^2}{\omega}-\sqrt{q^2-\lambda^2}}\right|\right]\Bigg\} .$$

Real corrections

$0 < \lambda < \omega$

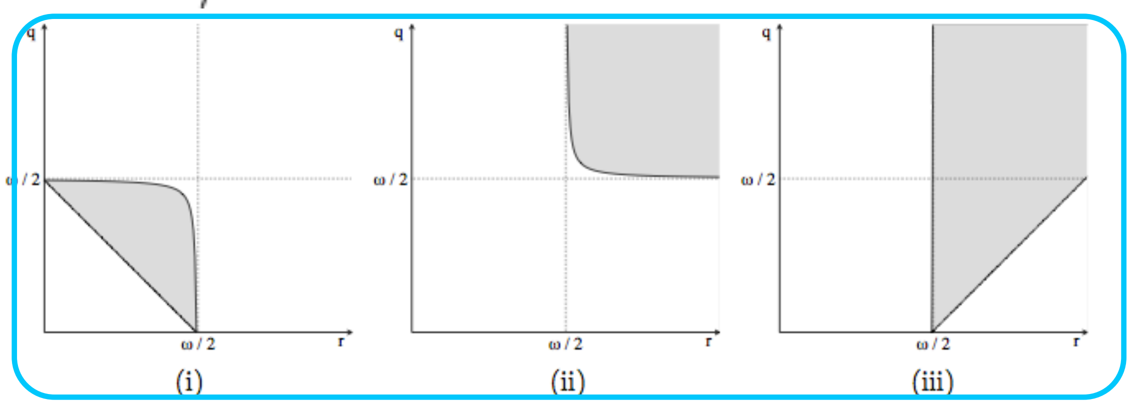
$$
\begin{aligned}
\rho^{(\mathrm{ps})}_{\mathcal{I}^0_j}(\omega) \;\equiv\; & \int_{\mathbf{q},\mathbf{r}} \frac{\omega^6 \pi}{4 q r E_{qr}} \Big\{ \\
&+ \Big[\delta(\omega - q - r - E_{qr}) - \delta(\omega + q + r + E_{qr})\Big] \frac{(1+n_{qr})(1+n_q+n_r)+n_q n_r}{(q+r+E_{qr})^2(q-r+E_{qr})(q-r-E_{qr})} \\
&+ \Big[\delta(\omega - q - r + E_{qr}) - \delta(\omega + q + r - E_{qr})\Big] \frac{n_{qr}(1+n_q+n_r)-n_q n_r}{(q+r-E_{qr})^2(q-r+E_{qr})(q-r-E_{qr})} \\
&+ \Big[\delta(\omega - q + r - E_{qr}) - \delta(\omega + q - r + E_{qr})\Big] \frac{n_r(1+n_q+n_{qr})-n_q n_{qr}}{(q-r+E_{qr})^2(q+r+E_{qr})(q+r-E_{qr})} \\
&+ \Big[\delta(\omega + q - r - E_{qr}) - \delta(\omega - q + r + E_{qr})\Big] \frac{n_q(1+n_r+n_{qr})-n_r n_{qr}}{(q-r-E_{qr})^2(q+r+E_{qr})(q+r-E_{qr})} \Big\} \, .
\end{aligned}
$$

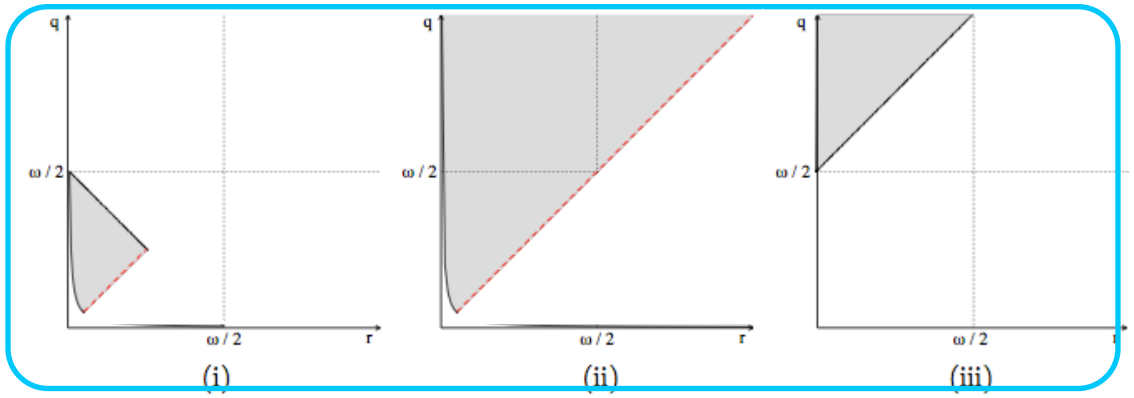
$$
\boxed{
\begin{aligned}
(1+n_{qr})(1+n_q+n_r)+n_q n_r &= n_q n_r n_{qr}\left(e^{q+r+E_{qr}}-1\right) , \\
n_{qr}(1+n_q+n_r)-n_q n_r &= n_q n_r n_{qr}\left(e^{q+r}-e^{E_{qr}}\right) , \\
n_r(1+n_q+n_{qr})-n_q n_{qr} &= n_q n_r n_{qr}\left(e^{q+E_{qr}}-e^{r}\right) , \\
n_q(1+n_r+n_{qr})-n_r n_{qr} &= n_q n_r n_{qr}\left(e^{r+E_{qr}}-e^{q}\right) ,
\end{aligned}
}
$$

$$
\begin{aligned}
\rho^{(\mathrm{ps})}_{\mathcal{I}_j}(\omega) \;\equiv\; & \frac{2\omega^4}{(4\pi)^3} \int_0^\infty \mathrm{d}q \int_0^\infty \mathrm{d}r \int_{E^-_{qr}}^{E^+_{qr}} \mathrm{d}E_{qr}\, n_q n_r n_{qr} \Big\{ \\
\text{(i)} \quad & \frac{\delta(\omega - q - r - E_{qr})}{(2r-\omega)(2q-\omega)} \left(1 - e^{q+r+E_{qr}}\right) \\
\text{(ii)} \quad + & \frac{\delta(\omega - q - r + E_{qr})}{(2r-\omega)(2q-\omega)} \left(e^{E_{qr}} - e^{q+r}\right) \\
\text{(iii)} \quad + & \frac{\delta(\omega + q - r - E_{qr})}{(2r-\omega)(2q+\omega)} \left(e^{r+E_{qr}} - e^{q}\right) \\
\text{(iv)} \quad + & \frac{\delta(\omega - q + r - E_{qr})}{(2r+\omega)(2q-\omega)} \left(e^{q+E_{qr}} - e^{r}\right) \Big\} \, .
\end{aligned}
$$

Real corrections

$\lambda = \omega/10$



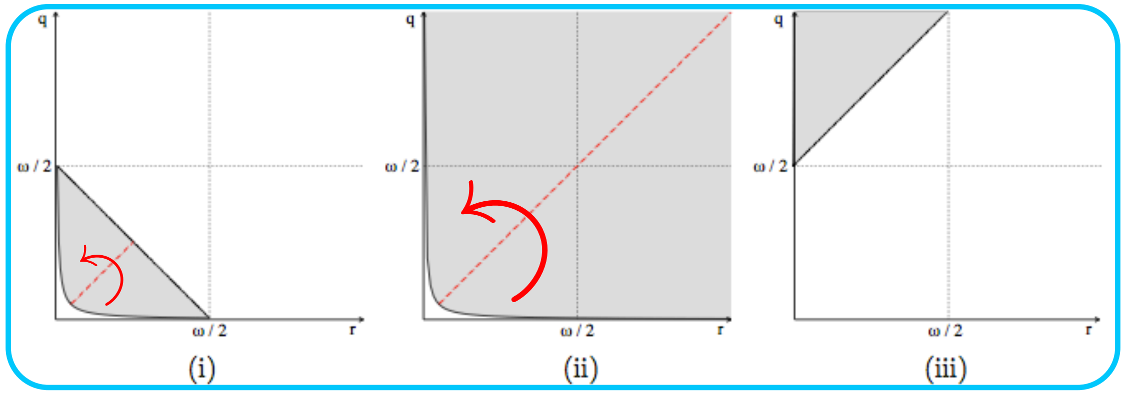


$$
\begin{aligned}
&\text{(i)}: && q \to \frac{\omega}{2} - q\,, \quad r \to \frac{\omega}{2} - r\,,\\
&\text{(ii)}: && q \to \frac{\omega}{2} + q\,, \quad r \to \frac{\omega}{2} + r\,,\\
&\text{(iii)}: && q \to -\frac{\omega}{2} + q\,, \quad r \to \frac{\omega}{2} + r\,,
\end{aligned}
$$

$$\text{(i)} \quad n_{\frac{\omega}{2}-q}\, n_{\frac{\omega}{2}-r}\, n_{q+r}(1-e^{\omega}) = -(1+2n_{\frac{\omega}{2}})\left[1 + n_{q+r} + n_{\frac{\omega}{2}-q} + (1+n_{\frac{\omega}{2}-r})\frac{n_{q+r}n_{\frac{\omega}{2}-q}}{n_r^2}\right].$$

$$\text{(ii)} \quad n_{\frac{\omega}{2}+q}\, n_{\frac{\omega}{2}+r}\, n_{q+r}\, e^{q+r}(1-e^{\omega}) = (1+2n_{\frac{\omega}{2}})\left[-n_{q+r} + n_{q+\frac{\omega}{2}} - (1+n_{q+\frac{\omega}{2}})\frac{n_{q+r}n_{r+\frac{\omega}{2}}}{n_r^2}\right].$$

$$\text{(iii)} \quad n_{q-\frac{\omega}{2}}\, n_{\frac{\omega}{2}+r}\, n_{q-r}\, e^{q-\frac{\omega}{2}}(e^{\omega}-1) = (1+2n_{\frac{\omega}{2}})\left[n_{q-\frac{\omega}{2}} - n_q - n_{q-\frac{\omega}{2}}\frac{(1+n_{q-r})(n_q - n_{r+\frac{\omega}{2}})}{n_r n_{-\frac{\omega}{2}}}\right].$$



$\rho_{\mathcal{I}_j^0}$

- Collect every part together and simplify them in the limit $\lambda \ll \omega$,
- All the divergent terms cancel each other, we can set $\lambda \to 0$ in the end.

$$
\begin{aligned}
&\frac{(4\pi)^3 \rho_{\mathcal{I}_j^0}(\omega)}{\omega^4(1+2n_{\frac{\omega}{2}})} = \\
&\quad \int_0^{\frac{\omega}{4}} \mathrm{d}q\, n_q \left[\left(\frac{1}{q-\frac{\omega}{2}} - \frac{1}{q}\right)\ln\Big(1-\frac{2q}{\omega}\Big) - \frac{\frac{\omega}{2}}{q(q+\frac{\omega}{2})}\ln\Big(1+\frac{2q}{\omega}\Big)\right] \\
&+ \int_{\frac{\omega}{4}}^{\frac{\omega}{2}} \mathrm{d}q\, n_q \left[\left(\frac{2}{q-\frac{\omega}{2}} - \frac{1}{q}\right)\ln\Big(1-\frac{2q}{\omega}\Big) - \frac{\frac{\omega}{2}}{q(q+\frac{\omega}{2})}\ln\Big(1+\frac{2q}{\omega}\Big) - \frac{1}{q-\frac{\omega}{2}}\ln\Big(\frac{2q}{\omega}\Big)\right] \\
&+ \int_{\frac{\omega}{2}}^{\infty} \mathrm{d}q\, n_q \left[\left(\frac{2}{q-\frac{\omega}{2}} - \frac{2}{q}\right)\ln\Big(\frac{2q}{\omega}-1\Big) - \frac{\frac{\omega}{2}}{q(q+\frac{\omega}{2})}\ln\Big(1+\frac{2q}{\omega}\Big) + \Big(\frac{1}{q} - \frac{1}{q-\frac{\omega}{2}}\Big)\ln\Big(\frac{2q}{\omega}\Big)\right] \\
&+ \int_0^{\frac{\omega}{2}} \mathrm{d}q \int_0^{\frac{\omega}{4}-|q-\frac{\omega}{4}|} \mathrm{d}r \left(-\frac{1}{qr}\right) \frac{n_{\frac{\omega}{2}-q}\, n_{q+r}(1+n_{\frac{\omega}{2}-r})}{n_r^2} \\
&+ \int_{\frac{\omega}{2}}^{\infty} \mathrm{d}q \int_0^{q-\frac{\omega}{2}} \mathrm{d}r \left(-\frac{1}{qr}\right) \frac{n_{q-\frac{\omega}{2}}(1+n_{q-r})(n_q - n_{r+\frac{\omega}{2}})}{n_r n_{-\frac{\omega}{2}}} \\
&+ \int_0^{\infty} \mathrm{d}q \int_0^{q} \mathrm{d}r \left(-\frac{1}{qr}\right) \frac{(1+n_{q+\frac{\omega}{2}})n_{q+r}n_{r+\frac{\omega}{2}}}{n_r^2} + \mathcal{O}(\lambda \ln \lambda)\,.
\end{aligned}
$$

More about the shear channel

$$\mathcal{I}_{\mathrm{h}}^{0} \equiv \sumint_{Q,R} \frac{P^4}{Q^2R^2(Q-R)^2(R-P)^2} ,$$

$$\mathcal{I}_{\mathrm{h}}^{1} \equiv \sumint_{Q,R} \frac{P^2}{Q^2R^2(Q-R)^2(R-P)^2} P_T(Q) ,$$

$$\mathcal{I}_{\mathrm{h}}^{2} \equiv \sumint_{Q,R} \frac{P^2}{Q^2R^2(Q-R)^2(R-P)^2} P_T(R) ,$$

$$\mathcal{I}_{\mathrm{h}}^{3} \equiv \sumint_{Q,R} \frac{P^4}{Q^2R^4(Q-R)^2(R-P)^2} P_T(R) ,$$

$$\mathcal{I}_{\mathrm{h}}^{4} \equiv \sumint_{Q,R} \frac{P^2}{Q^2R^4(Q-R)^2(R-P)^2} P_T(Q)^2 ,$$

$$\mathcal{I}_{\mathrm{h}}^{5} \equiv \sumint_{Q,R} \frac{P^2}{Q^2R^4(Q-R)^2(R-P)^2} P_T(R)^2 ,$$

$$\mathcal{I}_{\mathrm{h}}^{6} \equiv \sumint_{Q,R} \frac{P^2}{Q^2R^4(Q-R)^2(R-P)^2} P_T(Q)P_T(R) ,$$

$$\mathcal{I}_{\mathrm{h}}^{7} \equiv \sumint_{Q,R} \frac{P^2}{Q^2R^4(Q-R)^2(R-P)^2} P_T(Q)P_T(Q-R) ,$$

$$P_T(Q) \equiv Q_\mu Q_\nu P^T_{\mu\nu}(P) = \mathbf{q}^2 - (\mathbf{q}\cdot\hat{\mathbf{p}})^2$$

More about the shear channel

$$P_T(Q) \equiv Q_\mu Q_\nu P^T_{\mu\nu}(P) = \mathbf{q}^2 - (\mathbf{q}\cdot\hat{\mathbf{p}})^2$$

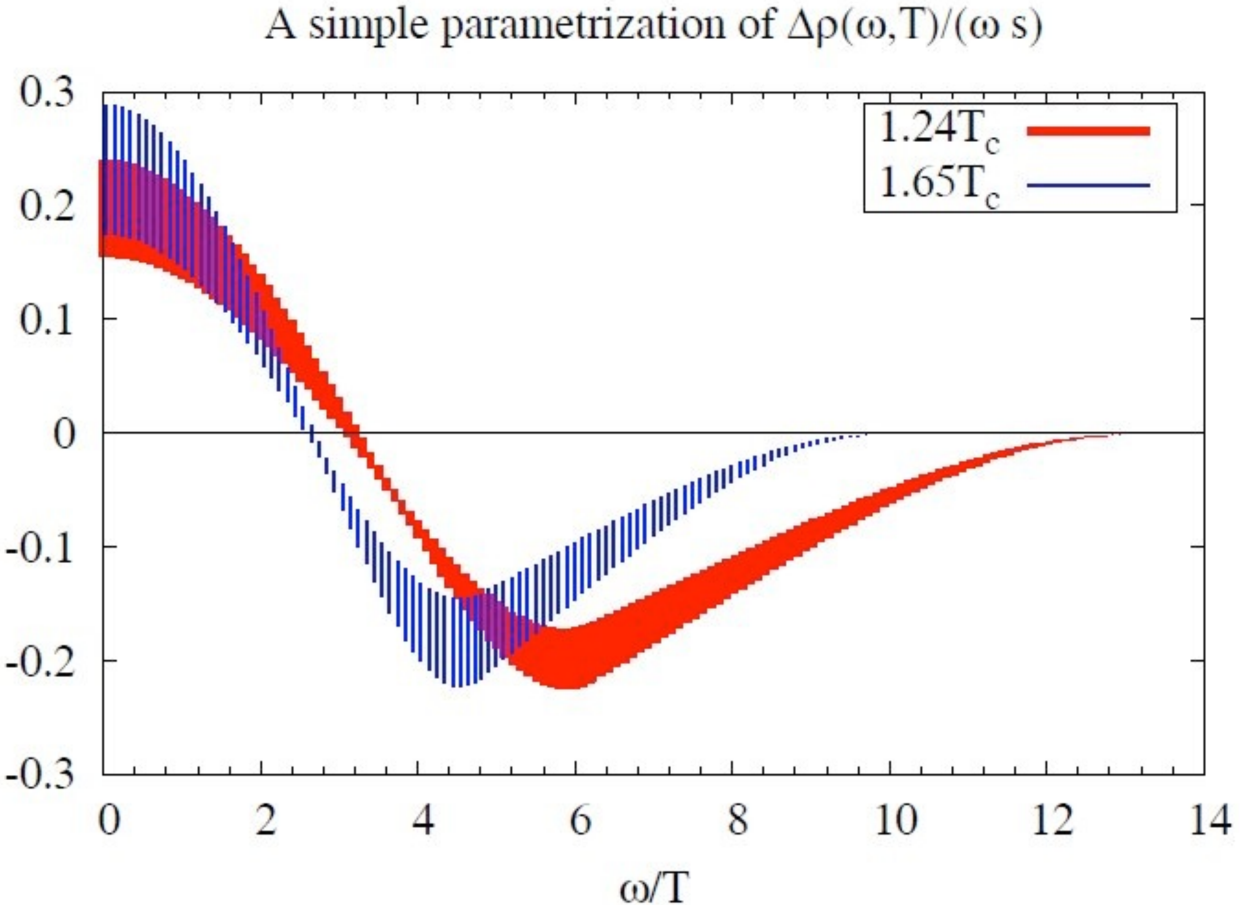
$$P_T(Q) \to \frac{D-2}{D-1} q^2 \,, \qquad P_T^2(Q) \to \frac{D(D-2)}{D^2-1} q^4$$

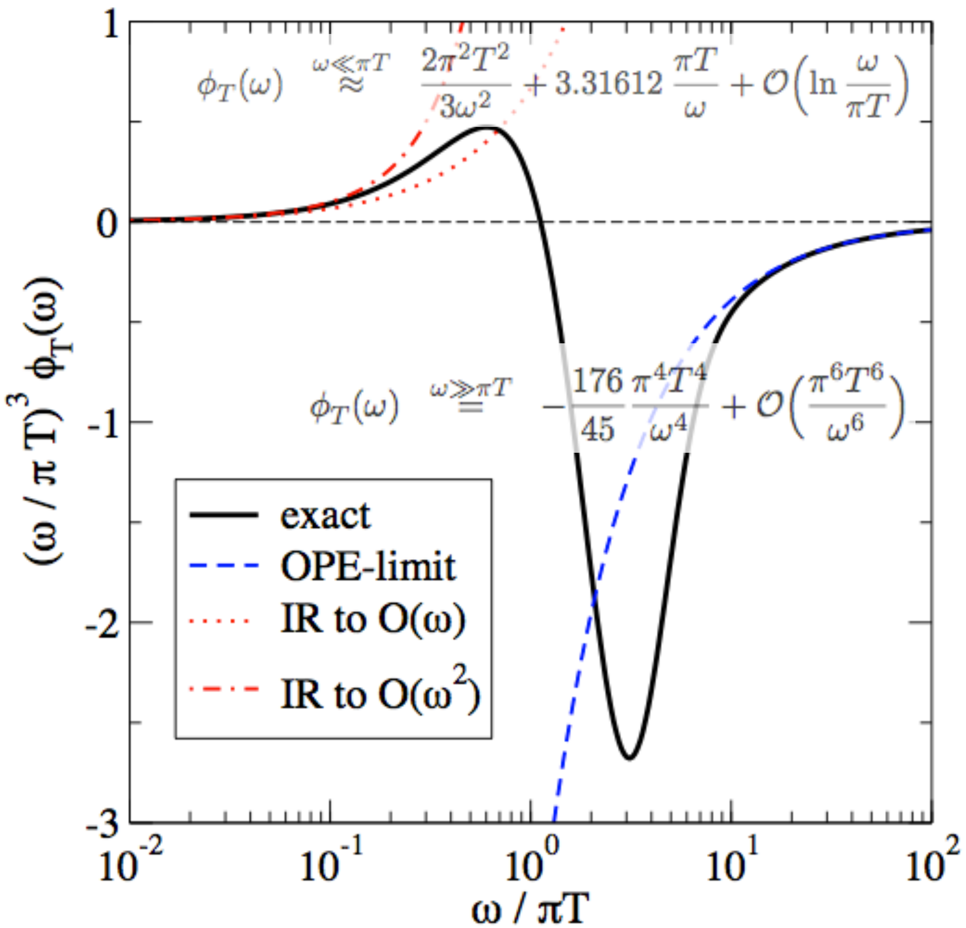
$$P_T(Q) P_T(R) \to \frac{D^2-2D-2}{D^2-1} q^2 r^2 + \frac{2}{D^2-1} (\mathbf{q}\cdot\mathbf{r})^2$$

$$\frac{1}{R^4} = -\lim_{m\to 0} \left\{ \frac{\mathrm{d}}{\mathrm{d}m^2} \frac{1}{R^2+m^2} \right\}$$

Spectral functions: Bulk channel

H. B. Meyer, 1002.3343

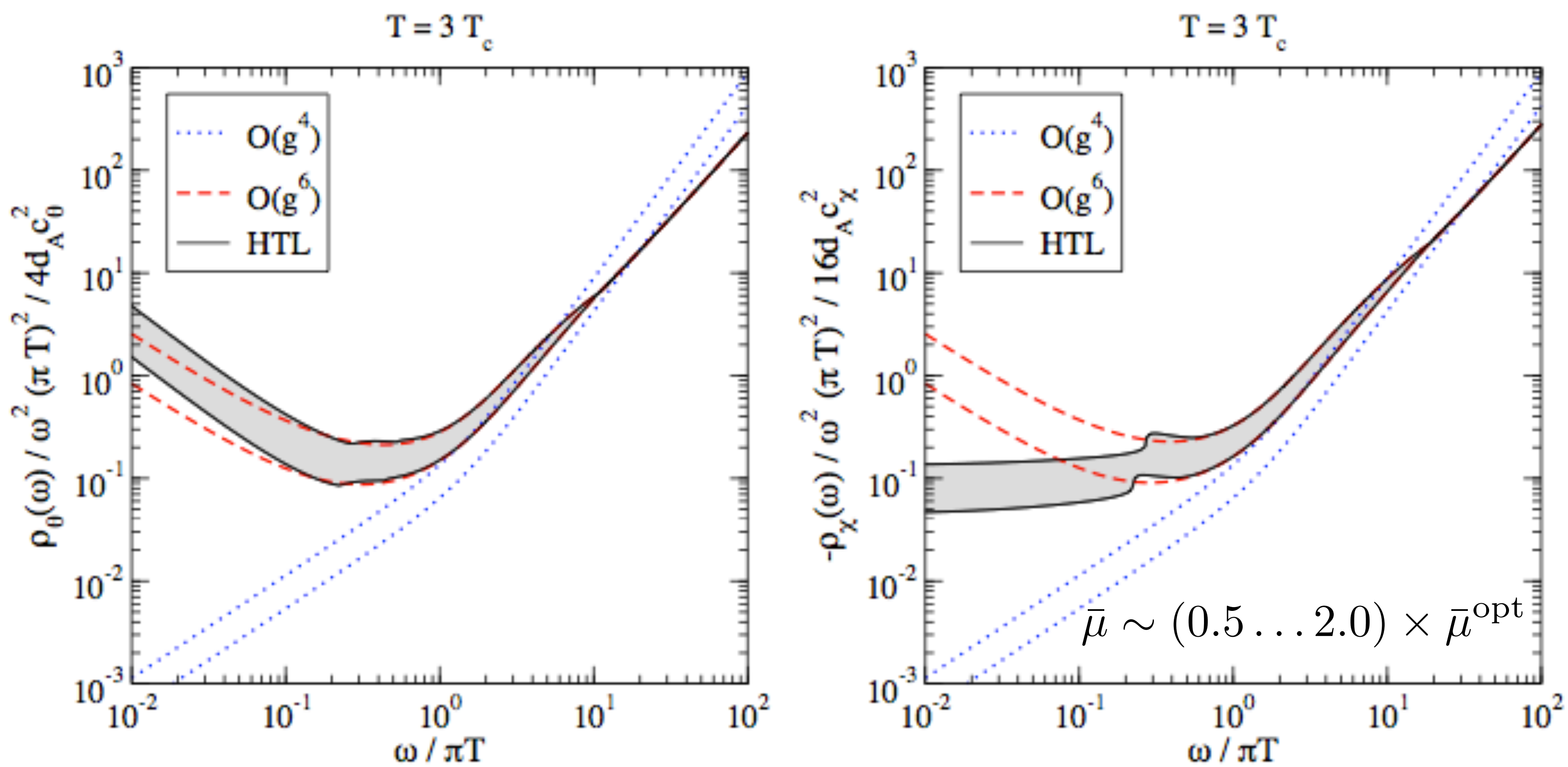




$$\frac{\rho_\theta(\omega)}{4 d_A c_\theta^2} = \frac{\pi\omega^4}{(4\pi)^2}\left(1+2n_{\frac{\omega}{2}}\right)\left\{g^4 + \frac{g^6 N_c}{(4\pi)^2}\left[\frac{22}{3}\ln\frac{\bar{\mu}^2}{\omega^2} + \frac{73}{3} + 8\,\phi_T(\omega)\right]\right\} + \mathcal{O}(g^8)$$

$$\frac{-\rho_\chi(\omega)}{16 d_A c_\chi^2} = \frac{\pi\omega^4}{(4\pi)^2}\left(1+2n_{\frac{\omega}{2}}\right)\left\{g^4 + \frac{g^6 N_c}{(4\pi)^2}\left[\frac{22}{3}\ln\frac{\bar{\mu}^2}{\omega^2} + \frac{97}{3} + 8\,\phi_T(\omega)\right]\right\} + \mathcal{O}(g^8)$$

Spectral functions: Bulk channel

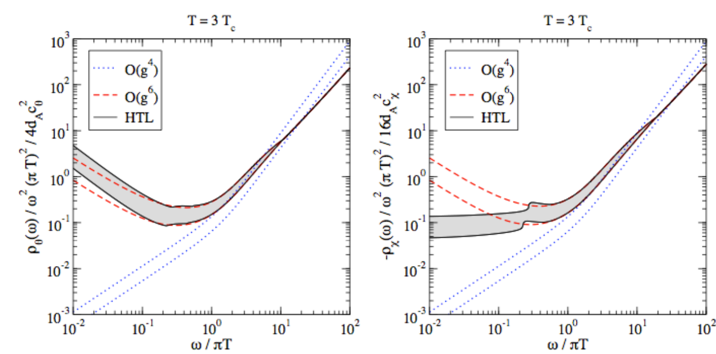


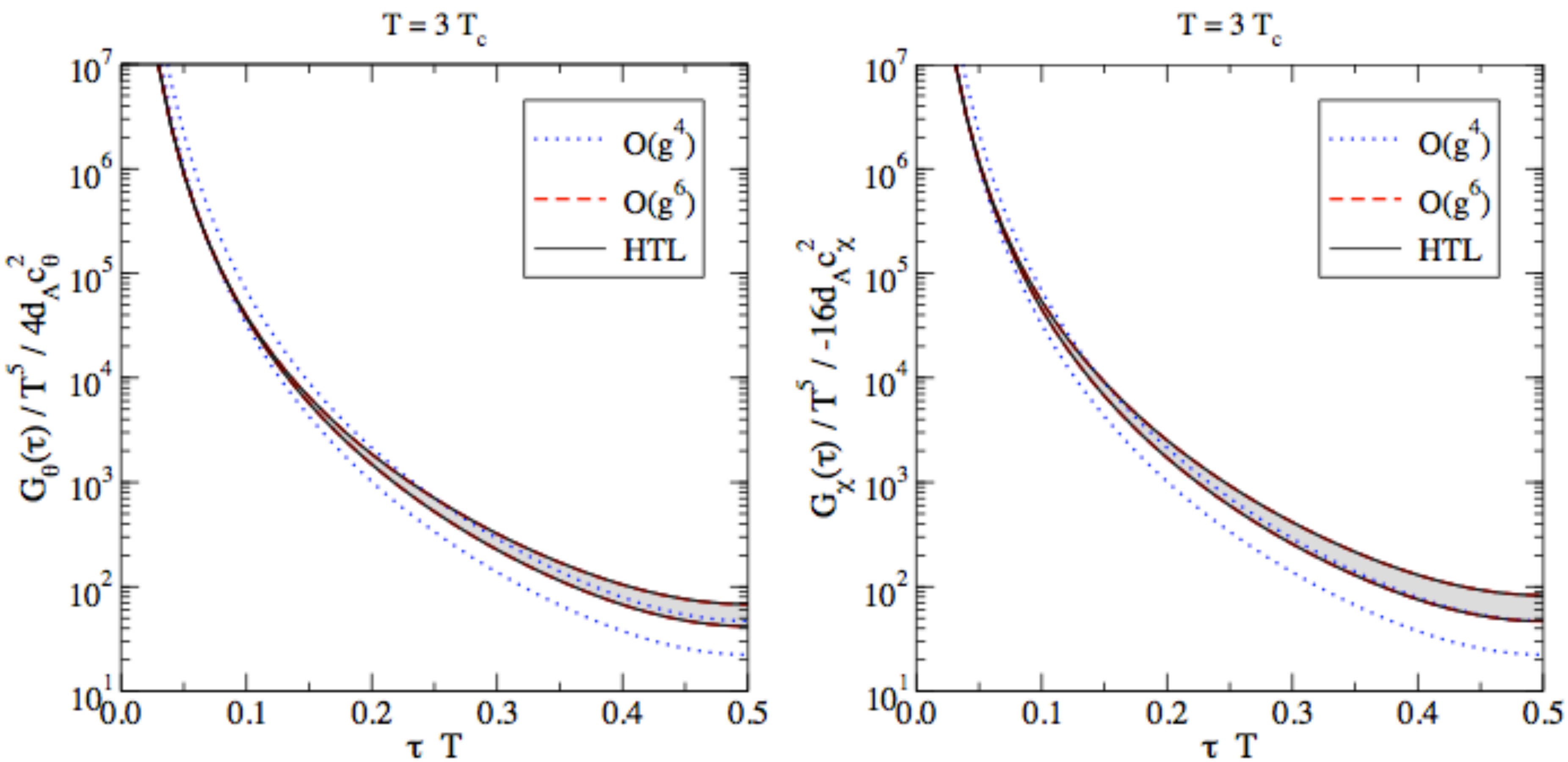
$\bar{\mu} \sim (0.5 \ldots 2.0) \times \bar{\mu}^{\rm opt}$

$$\rho^{\rm QCD}_{\rm resummed} = \rho^{\rm QCD}_{\rm resummed} - \rho^{\rm HTL}_{\rm resummed} + \rho^{\rm HTL}_{\rm resummed} \approx \rho^{\rm QCD}_{\rm naive} - \rho^{\rm HTL}_{\rm naive} + \rho^{\rm HTL}_{\rm resummed} \, .$$

Imaginary-time correlators: Bulk channel

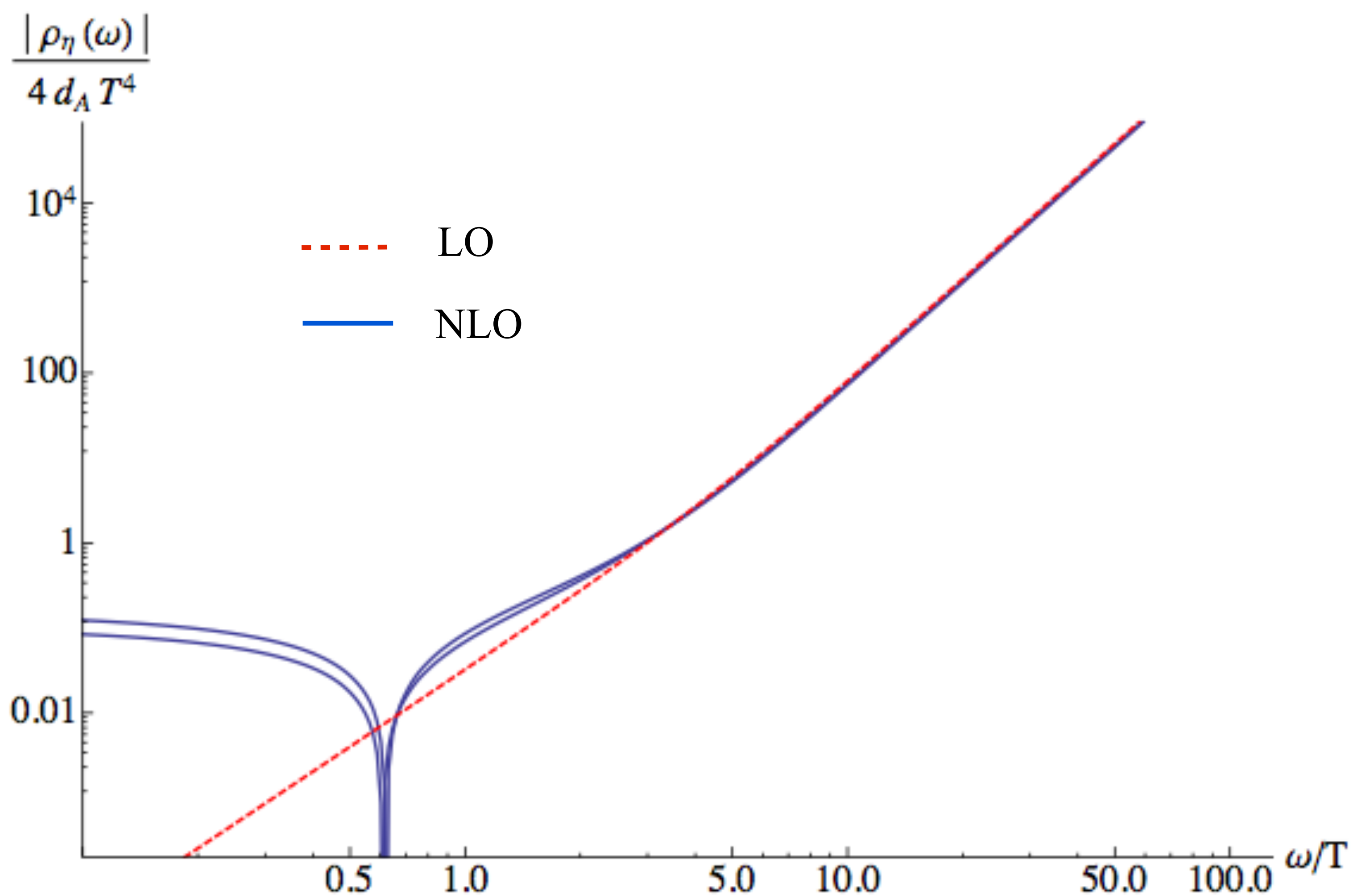
$$G(\tau) = \int_0^\infty \frac{d\omega}{\pi}\rho(\omega,\mathbf{0})\frac{\cosh\left(\frac{\beta}{2}-\tau\right)\omega}{\sinh\frac{\beta\omega}{2}} \,.$$





- Considerable difference between LO and NLO in spectral function leads to a small correction to the imaginary-time correlators.

Spectral functions: Shear channel



$$\frac{\rho_\eta(\omega)}{4d_A} = \frac{\omega^4}{4\pi}(1+2n_{\frac{\omega}{2}})\left\{-\frac{1}{10}+\frac{g^2N_c}{(4\pi)^2}\left(\frac{2}{9}+\phi_T^\eta(\omega/T)\right)\right\}$$

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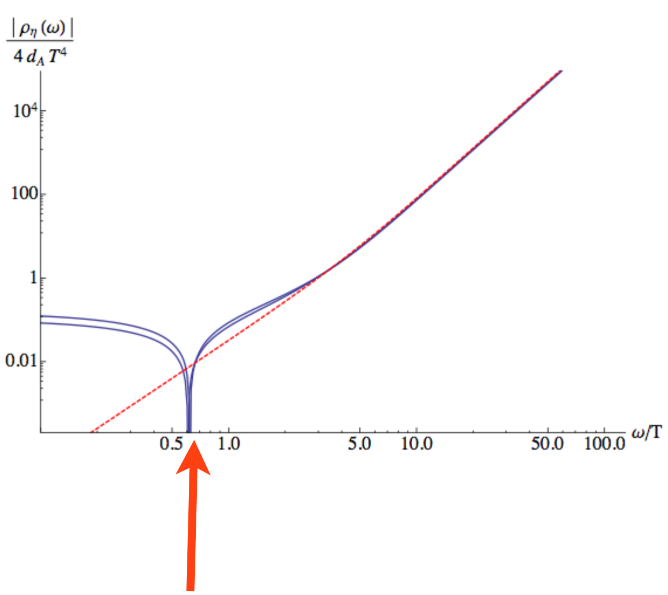
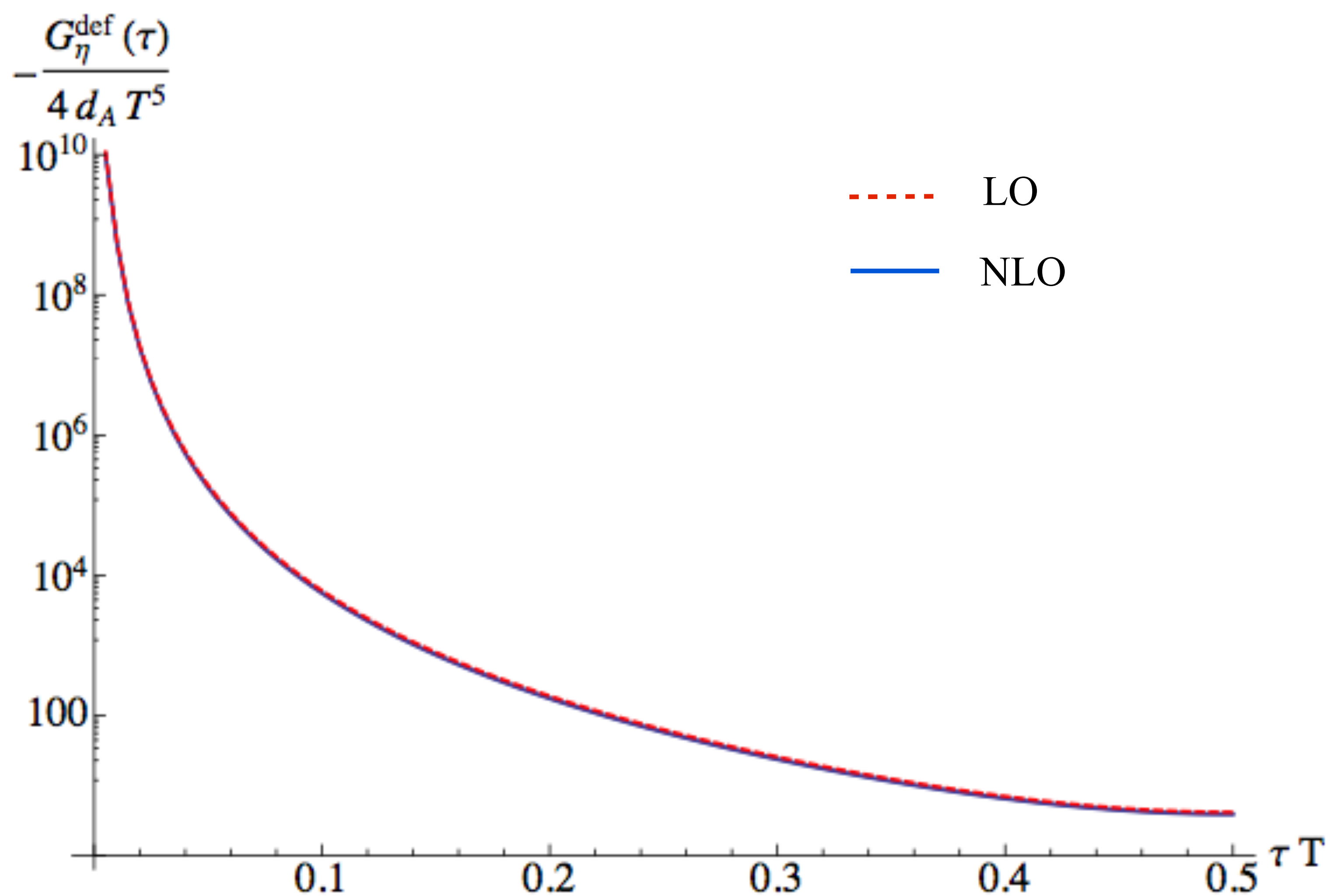
Imaginary-time correlators: Shear channel

$$G_\eta^{\mathrm{def}}(\tau) = \int_{\omega_0}^{\infty} \frac{\mathrm{d}\omega}{\pi} \rho_\eta(\omega) \frac{\cosh\left[\left(\frac{\beta}{2} - \tau\right)\omega\right]}{\sinh\frac{\beta\omega}{2}}$$

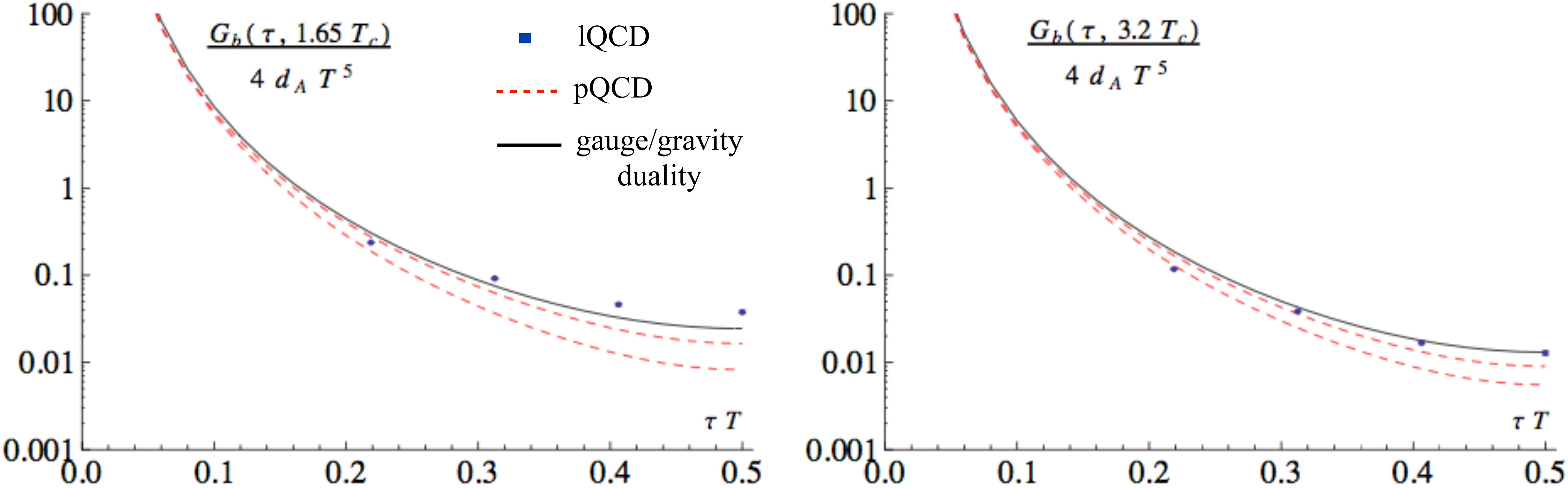


$\omega_0 \approx 0.6T$



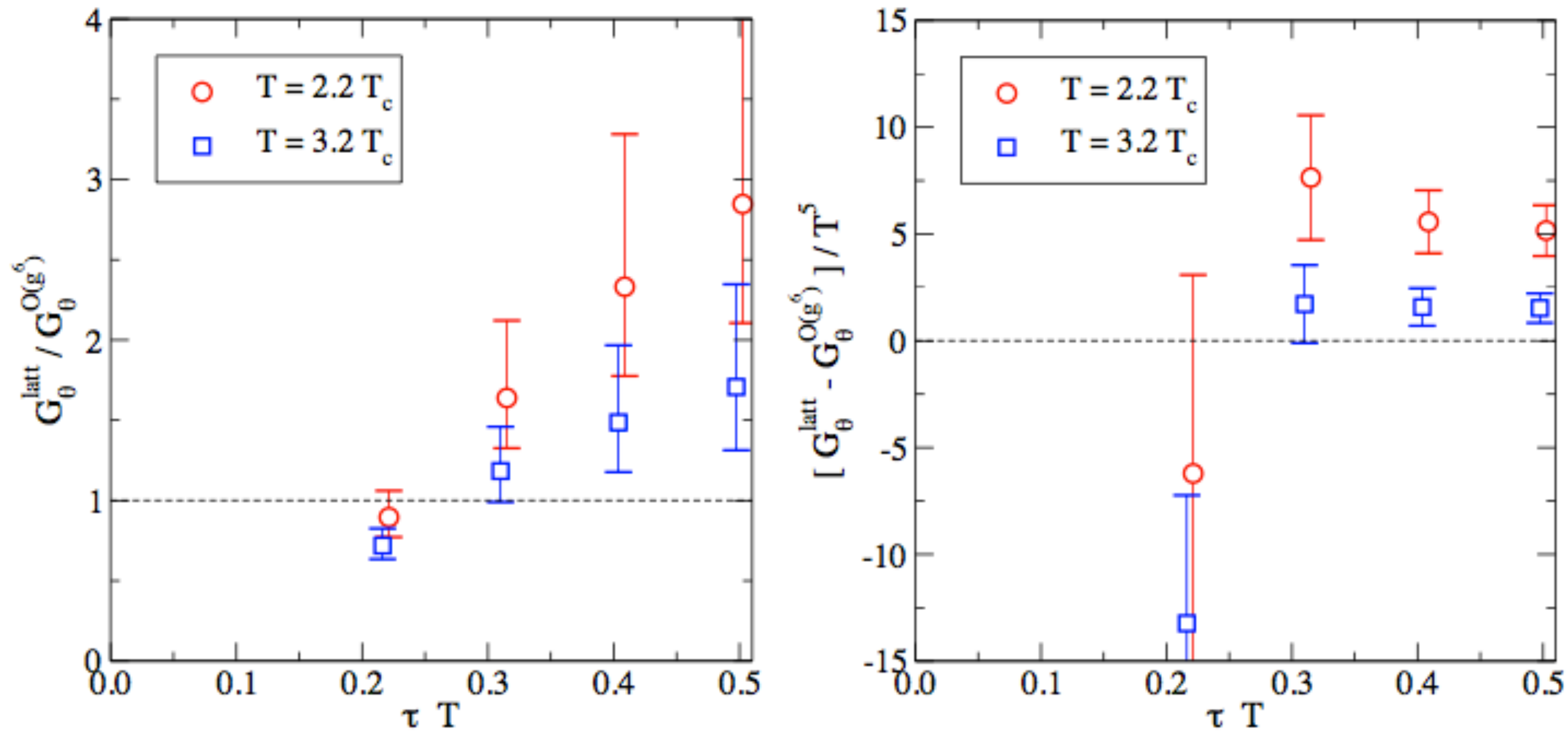
Lattice vs. pQCD vs. gauge/gravity duality: Bulk channel

K. Kajantie, M. Krssak and A. Vuorinen, arXiv:1302.1432 [hep-ph].



Lattice vs. pQCD: Bulk channel

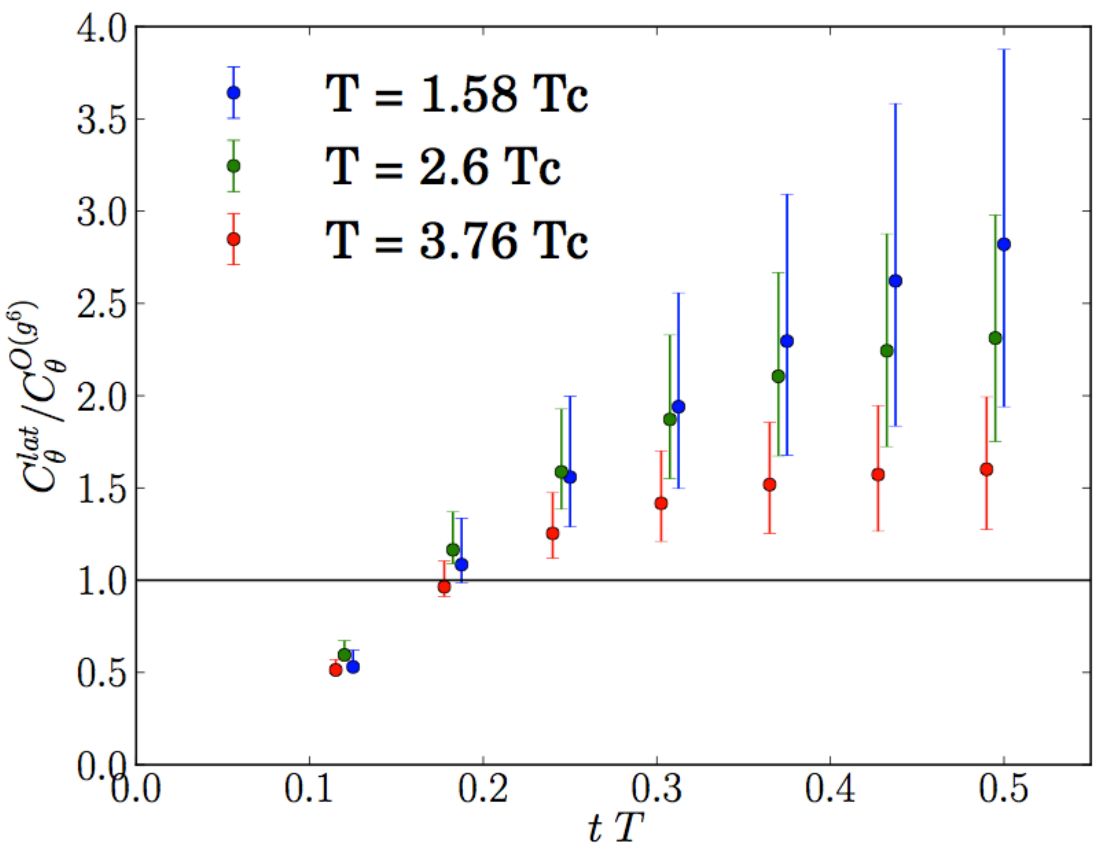
Lattice data from H.B. Meyer, JHEP 04(2010), 099 [10023344]



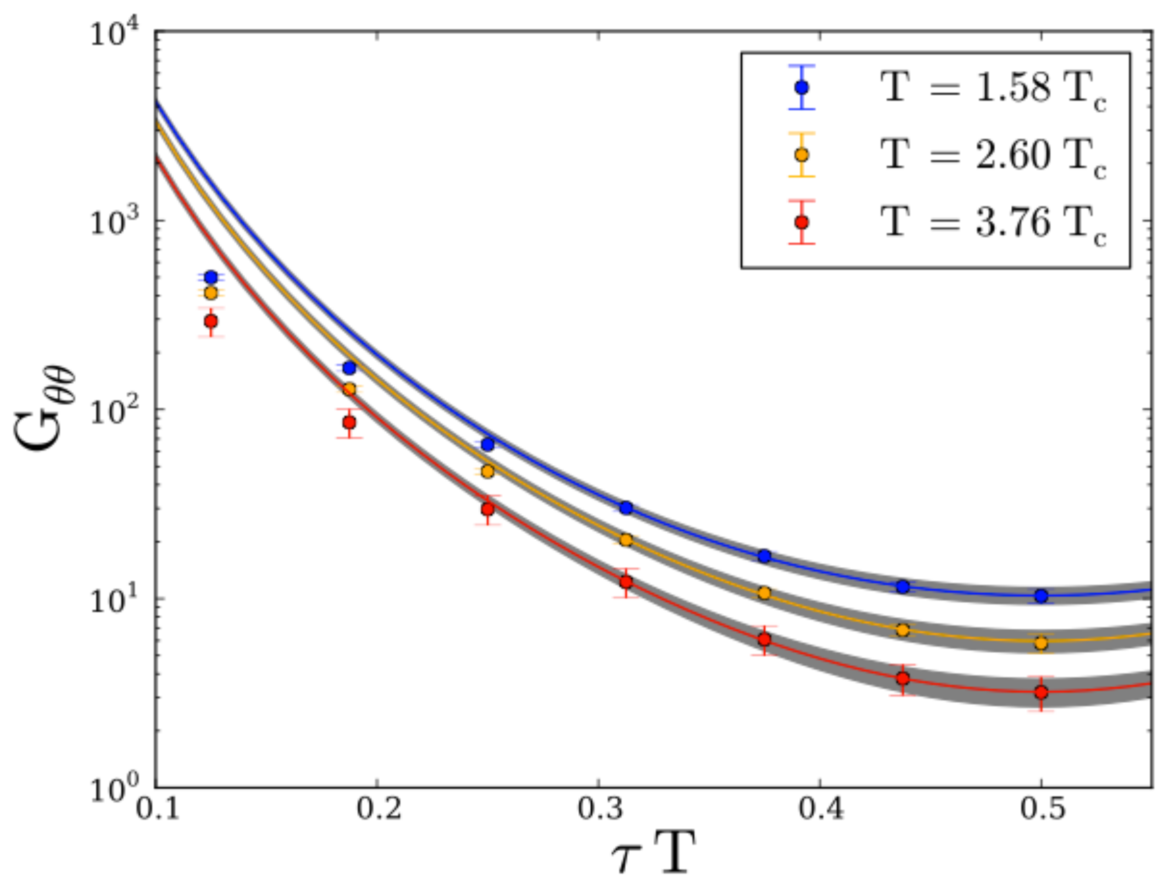
- The ratio shows good agreement at short distances.
- The difference no longer shows the short distance divergence.
  A model independent analytic continuation could be attempted.

Lattice vs. pQCD: Bulk channel

Chuan Miao(CPOD2011), H. B. Meyer



Chuan Miao, H.B. Meyer (Preliminary)



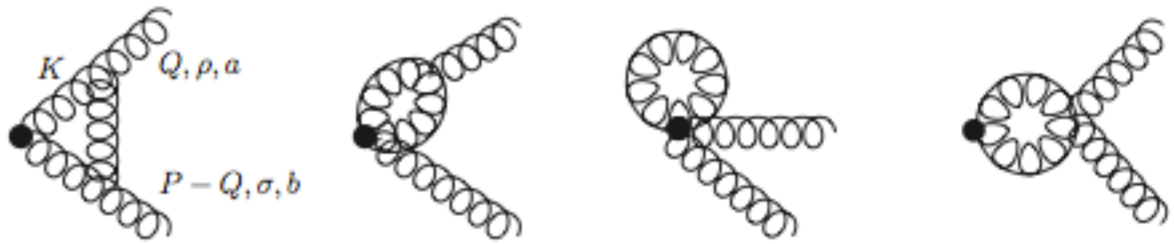
- Right panel: Fit correlators with Breit Wigner formula (low frequency) + NLO results (high frequency), the width of the B-W is fixed to 0.5πT.
- NLO perturbative input is very helpful.

HTL Propagator & Vertex

- HTL propagator:

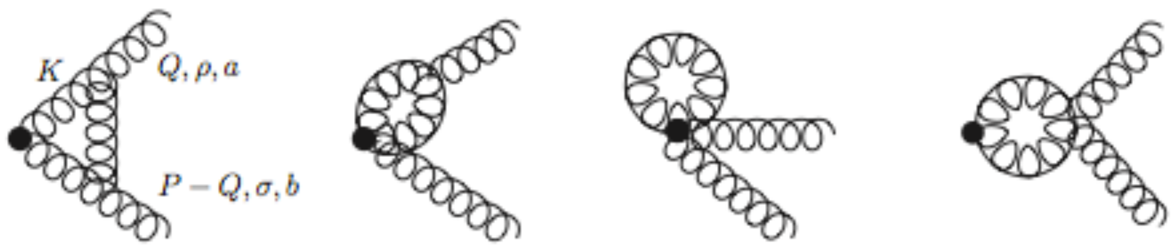
$$\langle A^a_\mu(X)\, A^b_\nu(Y)\rangle = \delta^{ab} \sum\!\!\!\!\!\!\int_Q e^{iQ\cdot(X-Y)}\left[\frac{\mathbb{P}^T_{\mu\nu}(Q)}{Q^2+\Pi_T(Q)} + \frac{\mathbb{P}^E_{\mu\nu}(Q)}{Q^2+\Pi_E(Q)} + \frac{\xi\, Q_\mu Q_\nu}{Q^4}\right]$$

- HTL Vertex:



$$\left[V^{\theta/\chi}_{\mathrm{HTL}}\right]^{ab}_{\rho,\sigma} = 0$$

HTL Vertex in Shear Channel



$$\tilde{G}_\eta(P) \equiv 2X_{\mu\nu,\alpha\beta}\,\tilde{G}_{\mu\nu,\alpha\beta}(P)\,,$$

$$\begin{aligned}
\frac{\left[V^\eta_{\mathrm{HTL}}\right]^{ab}_{\mu\nu,\rho\sigma}}{g_{\mathrm{B}}^2 N_{\mathrm{c}}} = & \sum\!\!\!\!\!\!\!\int_K \frac{\delta_{ab}A^a_\rho(Q)A^b_\sigma(P-Q)}{K^2(K-P)^2(K-Q)^2}\Big\{4(2-D)K_\mu K_\nu K_\rho K_\sigma \\
& - 2(2-D)K_\mu K_\nu(K_\rho Q_\sigma + Q_\rho K_\sigma + K_\rho P_\sigma) - 4\Big[(3-D)K_\mu P_\nu - P_\mu K_\nu\Big]K_\rho K_\sigma\Big\} \\
& + \sum\!\!\!\!\!\!\!\int_K \frac{1}{K^2(K-P)^2}\Big\{\delta_{\mu\rho}K_\nu K_\sigma - \delta_{\mu\sigma}K_\nu K_\rho + \delta_{\nu\sigma}K_\mu K_\rho - \delta_{\nu\rho}K_\mu K_\sigma \\
& + (D-2)K_\mu K_\nu \delta_{\rho\sigma}\Big\} + \sum\!\!\!\!\!\!\!\int_K \frac{1}{K^2(K-Q)^2}\Big\{\delta_{\mu\rho}K_\nu K_\sigma - (3-2D)\delta_{\mu\sigma}K_\nu K_\rho \\
& - \delta_{\nu\sigma}K_\mu K_\rho - \delta_{\nu\rho}K_\mu K_\sigma\Big\} + \sum\!\!\!\!\!\!\!\int_K \frac{1}{(K-P)^2(K-Q)^2}\Big\{-(6-2D)\delta_{\mu\sigma}K_\nu K_\rho \\
& + 2\delta_{\nu\sigma}K_\mu K_\rho\Big\} + \sum\!\!\!\!\!\!\!\int_K \frac{1}{K^2}\Big\{(3-D)\delta_{\mu\rho}\delta_{\nu\sigma}\Big\}
\end{aligned}$$

HTL Correction to Correlators

- With HTL propagator, naive HTL correlator in bulk channel completely matches IR limit of naive QCD.
- If $\left.\frac{\rho_\eta^{\mathrm{HTL}}(\omega)}{4d_A}\right|_{\mathrm{naive}} = \frac{1}{4\pi}(1+2n_{\frac{\omega}{2}})\left\{-\frac{\omega^4}{10}+\frac{\omega\pi^2 T}{45}m_E^2\right\}$, unfortunately, when only HTL propagator involved,

$$\left.\frac{\rho_\eta^{\mathrm{HTL}}(\omega)}{4d_A}\right|_{\mathrm{naive}} = \frac{1}{4\pi}(1+2n_{\frac{\omega}{2}})\left\{-\frac{\omega^4}{10}-\frac{\omega\pi^2 T}{45}m_E^2\right\}$$

- Naive HTL contribution from HTL vertex should be 2 x $\frac{1}{4\pi}(1+2n_{\frac{\omega}{2}})\frac{\omega\pi^2 T}{45}m_E^2$,but we get 0 x $\frac{1}{4\pi}(1+2n_{\frac{\omega}{2}})\frac{\omega\pi^2 T}{45}m_E^2$.
- More than 4000 terms in full HTL correlator are waiting.

Summary and Outlook

- Information on correlation functions of the energy momentum tensor crucial for disentangling the properties of the QGP
  - Wilson coefficients refined and determined in the OPE
  - Spectral densities needed in extracting transport coefficients from lattice QCD data
- NLO results in the bulk and shear channels completed, HTL for the shear channel underway
  - Results promising, but quantitative comparisons await
- If pure YM results useful, inclusion of fermions straightforward